*Journal of Engineering Physics and Thermophysics, Vol. 80, No. 5, 2007*

# DISTINCTIVE FEATURES OF FILTRATION IN A GRANULAR BED NEAR THE WALL

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UDC 532.5

*A generalization of the Brinkman equation to the case of linear filtration has been proposed based on an analysis of the existing models of filtration of a liquid (gas) in a stationary granular bed. It has been shown that, with allowance for the local-porosity distribution over the cross section of the apparatus, this equation makes it possible to describe velocity distributions observed experimentally in the wall zone.*

**Introduction.** Liquid (gas) flow in a granular bed near the wall is known to possess a number of distinctive features [1, 2]. Of them, the main one is a pronounced increase in the local velocity. A combination of properties of wall flow, which is related to the existence of the increased-velocity zone, is called the bypassing effect or the wall effect in the literature. In most cases this phenomenon impairs the operation of granular-bed apparatuses, since motion with an off-design velocity occurs in the wall region and an additional thermal resistance is created [3]. The character of change in the local velocity is determined, in many respects, by the ordering action of the wall on the structure of adjacent particle layers. This results in a special distribution of the local velocity in the wall region, which is also related to the method of charging a dispersed material, particle shape, dimension of the apparatus, and others. In all cases we have a velocity maximum near the wall on a linear scale of the order of the particle diameter.

The influence of the above factors on wall flow has mainly been investigated in the regimes of linear filtration [1]. The present work seeks to establish a relationship between the wall velocity of the liquid (gas) and the local-porosity distribution and to develop a procedure for calculating the profiles of this velocity in different regimes of flow (including nonlinear filtration).

**Local-Porosity Distribution Near the Wall.** Random packings near the wall are known [1, 2] to be characterized by increased porosity values related to its ordering influence. In [1], it was proposed that the porosity distribution be calculated from the formula

$$\varepsilon = \varepsilon_0 + \left(1 - \varepsilon_0 - \frac{\pi}{4}\left|\sin\frac{\pi y}{d}\right|\right)\exp\left(-0.65\,\frac{y}{d}\right) \tag{1}$$

on the condition that the first row of spherical particles has the porosity of a cubic packing. In [4], experimental data obtained were generalized, based on (1), for particles of a spherical shape

$$\varepsilon = \varepsilon_0 + \left(1 - \varepsilon_0 - \frac{\pi}{3.5}\left|\sin\frac{\pi y}{d}\right|\right)\exp\left(-\frac{y}{d}\right) \tag{2}$$

and an arbitrary shape

$$\varepsilon = \varepsilon_0 + \left(1 - \varepsilon_0 - \frac{\pi}{5}\left|\sin 0.8\,\frac{\pi y}{d}\right|\right)\exp\left(-2.3\,\frac{y}{d}\right). \tag{3}$$

Based on (1) and (2), we propose a simple dependence

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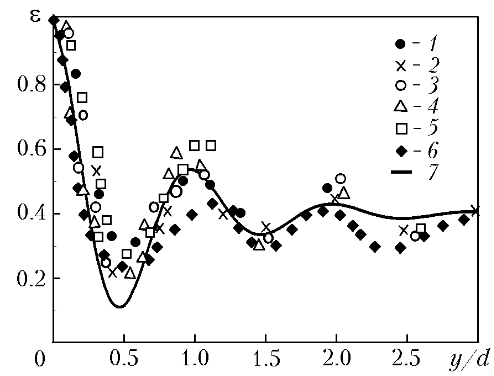


Fig. 1. Distribution of the local porosity of a bed of spherical particles near the wall: 1) $d$ = 0.72, 2) 0.82, 3) 1.3, 4) 1.7, 5) 2.2, 6) 2.6 (1–5) [4] and 6) [2]), and 7) (4). $y$, m.

$$\varepsilon = \varepsilon_0 + (1 - \varepsilon_0) \cos\left(2\frac{\pi y}{d}\right) \exp\left(-1.5\frac{y}{d}\right), \tag{4}$$

for a bed of spherical particles; it satisfactorily describes experimental data (Fig. 1) and represents a smooth function convenient for subsequent analysis.

**Models of Flow in the Granular Bed.** To describe flow in the granular bed one uses filtration theory based on Darcy's law which in the simplest case has the form

$$-\frac{\partial P}{\partial x} = \alpha \rho_f u . \tag{5}$$

The coefficient $\alpha$ is given by the expression [1]

$$\alpha = \alpha_1 + \alpha_2 u , \tag{6}$$

which determines different filtration regimes. When the flow velocities are small, we have $\alpha \approx \alpha_1$ = const and (5) becomes the equation of linear filtration

$$-\frac{\partial P}{\partial x} = \alpha_1 \rho_f u . \tag{7}$$

Under these conditions, use is also made of a very popular Brinkman model [5] which represents a generalization of Eq. (7):

$$-\frac{\partial P}{\partial x} = \alpha_1 \rho_f u - \mu_f \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right). \tag{8}$$

When the velocities are large, we have $\alpha \approx \alpha_2 u$ and (5) changes to the equation of nonlinear filtration

$$-\frac{\partial P}{\partial x} = \alpha_2 \rho_f u^2 . \tag{9}$$

In practice, (5) is frequently used in the form of the Ergun empirical formula [2]

$$-\frac{\partial P}{\partial x} = 150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu_f u}{d^2} + 1.75\frac{1-\varepsilon}{\varepsilon^3}\frac{\rho_f u^2}{d} , \tag{10}$$

well describing the pressure drop in granular beds in a wide range of experimental conditions. A comparison of (5) and (10) yields expressions for the coefficients $\alpha_1$ and $\alpha_2$:

$$\alpha_1 = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\nu_f}{d^2}; \quad \alpha_2 = 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{1}{d}, \tag{11}$$

these expressions have been used in the present work. We note that, with account for (11), the Brinkman equation (8) can be represented in the form

$$-\frac{\partial P}{\partial x} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu_f u}{d^2} - \mu_f \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right). \tag{12}$$

**Model of Flow in the Wall Zone.** To formulate such a model we must primarily consider the question of the value of the liquid (gas) velocity on a solid wall. The experimental data available in the literature do not provide an unambiguous answer to this question and only a comparative qualitative analysis of different filtration-flow models is possible at the moment.

The simplest assumption that $u(0) = 0$ is due to the influence of viscosity. However, with allowance for the fact that distances comparable to the particle diameter are eliminated from macroscopic consideration of Eqs. (5) and (7)–(10), the slip condition $u(0) = u_w \neq 0$ [1] is also acceptable on the wall. The latter is consistent with the fact that no sticking condition is set for the averaged motion on the particles within the bed. Clearly, in this case exterior walls are not fundamentally different from interior ones, i.e., particles.

Taking account of the comments made, we analyze the possibility of using Eqs. (5) and (7)–(10) for description of wall flow. The Brinkman equation (8) (or (12)) contains a viscous term and enables us to set the sticking or slip condition. A substantial drawback is that one can use it only under linear-filtration conditions. Equation (5) and its particular cases (7), (9), and (10) yield $u(0) \to \infty$ with account for $\varepsilon(0) = 1$ for a finite, nonzero, pressure gradient. For regularization of the problem we can use the method of averaging $\varepsilon(y)$ [1]. Another method of regularization is allowance for the liquid (gas) viscosity in these equations, which enables us to set the required boundary conditions. In the case of linear filtration this is implemented using the Brinkman equation (8) or (12). In the general case such a regularization of the problem can be carried out based on the equation [6]

$$-\frac{\partial P}{\partial x} = \alpha \rho_f u - \mu_f \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right),$$

which, with account for (11), will have the form

$$-\frac{\partial P}{\partial x} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu_f u}{d^2} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho_f u^2}{d} - \mu_f \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right). \tag{13}$$

As is seen, (13) represents a generalization of the Brinkman equation (12) to the case of nonlinear filtration by superposition of the Ergun equation (10) and the equation for viscous flow in the channel. For small velocities (linear filtration), it is reduced to the Brinkman equation (12), whereas for large velocities (nonlinear filtration) (13) takes the form

$$-\frac{\partial P}{\partial x} = 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho_f u^2}{d} - \mu_f \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right). \tag{14}$$

The structure of (14) yields that the influence of the viscous term must be pronounced in a narrow wall (boundary) layer where we have large velocity gradients because of the deceleration of the liquid. The main role of this term is to ensure regularization of flow for $y \to 0$. In the remaining part of the layer, (14) is equivalent to Eq. (9).

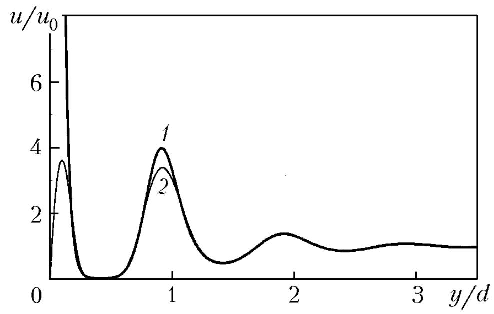


Fig. 2. Calculation of wall flow under linear-filtration conditions: 1) from the Darcy equation (7); 2) from the Brinkman equation (12).

**Calculation of Flow in the Wall Region.** Calculations of $u/u_0$ in the case of linear filtration were performed from the Darcy equation (7) and the Brinkman equation (12) with the sticking condition on the wall $u(0) = 0$. The value of the pressure gradient was calculated from the formula

$$-\frac{\partial P}{\partial x} = 150\,\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\,\frac{\mu_f u_0}{d^2}\,. \tag{15}$$

In accordance with this, the velocity profile, according to the Darcy equation, was determined by the expression

$$\frac{u}{u_0} = \frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\,\frac{\varepsilon^3}{(1-\varepsilon)^2}\,. \tag{16}$$

The velocity distribution in the case of the Brinkman equation (12) was found by numerical solution of the boundary-value problem

$$150\,\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3} - 150\,\frac{(1-\varepsilon)^2}{\varepsilon^3}\left(\frac{u}{u_0}\right) + \left(\frac{d}{R}\right)^2 \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial}{\partial r'}\left(\frac{u}{u_0}\right)\right) = 0\,, \tag{17}$$

$$\left.\frac{u}{u_0}\right|_{r'=1} = 0\,,\quad \left.\frac{\partial u}{\partial r'}\right|_{r'=0} = 0\,.$$

Figure 2 gives results of calculating the corresponding $u/u_0$ values. It is seen that the solutions of (16) and (17) are coincident everywhere, in practice, except for the narrow wall region of thickness of the order $d/4$. In the general case flow in the wall region was calculated within the framework of the Ergun equation (10) and the generalized Brinkman equation (13). The pressure gradient was prescribed in this case by a formula analogous to (15):

$$-\frac{\partial P}{\partial x} = 150\,\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\,\frac{\mu_f u_0}{d^2} + 1.75\,\frac{1-\varepsilon_0}{\varepsilon_0^3}\,\frac{\rho_f u_0^2}{d}\,. \tag{18}$$

The velocity profile determined by the Ergun equation was calculated from a formula yielded by (10):

$$\frac{u}{u_0} = \frac{150\,(1-\varepsilon_0)}{3.5\mathrm{Re}_0}\left(-1 + \sqrt{1 + \frac{7\varepsilon^3}{150^2\,(1-\varepsilon)^3}\left(150\,\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\,\mathrm{Re}_0 + 1.75\,\frac{1-\varepsilon_0}{\varepsilon_0^3}\,\mathrm{Re}_0^2\right)}\right). \tag{19}$$

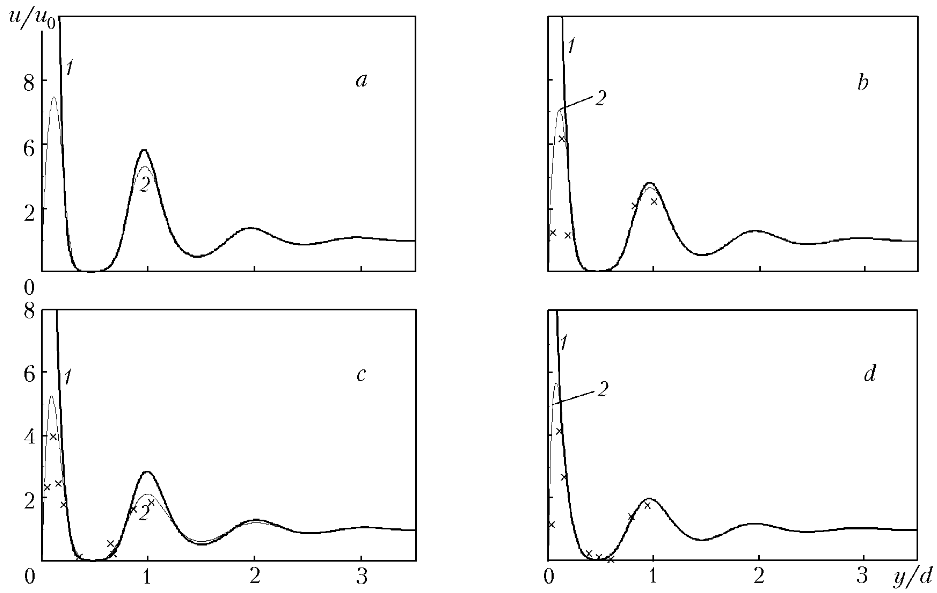


Fig. 3. Calculation of wall flow for different Reynolds numbers: a) $\mathrm{Re}_0$ = 1, b) 10, c) 50, and d) 100; 1) from the solution of the Ergun equation (19); 2) from the solution of the generalized Brinkman equation (20); points, experimental data [8].

The velocity distributions described by the generalized Brinkman equation (13) were found numerically by solution of the problem

$$150\,\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}+1.75\,\frac{1-\varepsilon_0}{\varepsilon_0^3}\,\mathrm{Re}_0-150\,\frac{(1-\varepsilon)^2}{\varepsilon^3}\left(\frac{u}{u_0}\right)-1.75\,\frac{1-\varepsilon}{\varepsilon^3}\,\mathrm{Re}_0\left(\frac{u}{u_0}\right)^2+$$

$$+\left(\frac{d}{R}\right)^2\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial}{\partial r'}\left(\frac{u}{u_0}\right)\right)=0\,, \tag{20}$$

$$\left.\frac{u}{u_0}\right|_{r'=1}=0\,,\quad \left.\frac{\partial u}{\partial r'}\right|_{r'=0}=0\,.$$

Figure 3 shows the wall-velocity profiles calculated from (19) and (20) for different $\mathrm{Re}_0$ numbers. As is seen, these solutions tend to converge with growth in $\mathrm{Re}_0$. As has been noted above, for large $\mathrm{Re}_0$ numbers, the solutions of (19) and (20) are coincident everywhere, except for the narrow wall zone of thickness (0.1–0.15)$d$.

Figure 4 gives results of analogous solutions of (19) and (20) with the slip condition on the wall. The slip velocity was taken to be equal to the slip velocity on solid particles in the bed's core, i.e., we had $u(0) = u_0$. For the sake of consistency, in this case we used the averaged values of local porosity

$$\langle\varepsilon\rangle=\frac{1}{kd}\int_y^{y+kd}\varepsilon\,(y)\,dy\,. \tag{21}$$

As is seen, the position of a velocity maximum second from the wall substantially shifted with growth in the averaging scale; the first maximum in the calculations from the generalized Brinkman equation (13) disappeared. A comparison of Figs. 3 and 4 yields that for $k$ = 0.1 the character of the velocity distribution is virtually independent

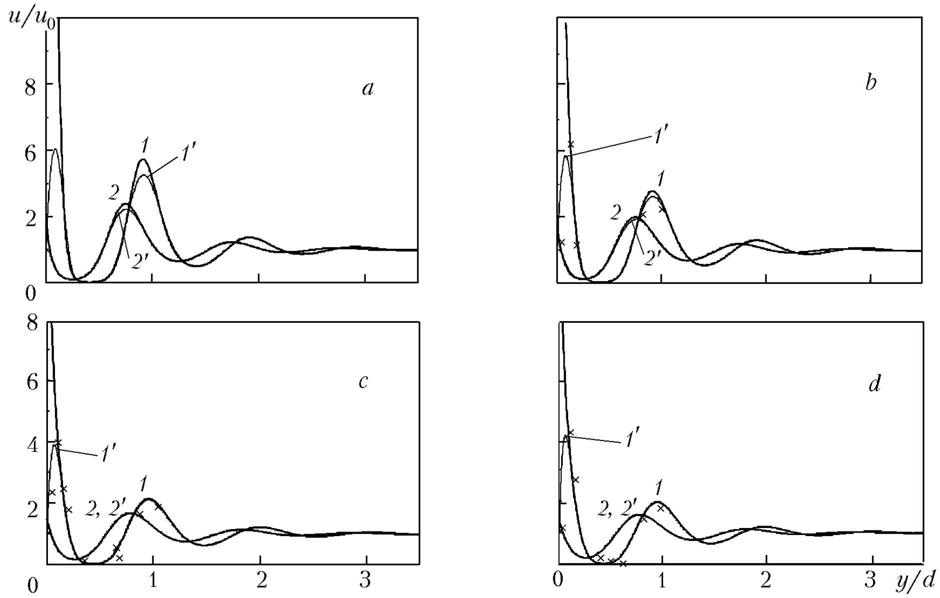


Fig. 4. Calculation of wall flow for different Reynolds numbers and the averaged porosity with the slip condition: a) $Re_0 = 1$, b) 10, c) 50, and d) 100; 1 and 2) from the solution of the Ergun equation (19); 1′ and 2′) from the solution of the generalized Brinkman equation (20); points, experimental data [8]. 1 and 1′) $k = 0.1$ and 2 and 2′) 0.5.

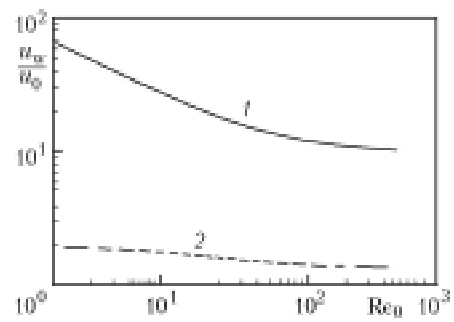


Fig. 5. Slip velocity according to the solution of the Ergun equation (19) for different scales of averaging of the porosity: 1) $k = 0.1$ and 2) 0.5.

of the type of boundary condition; it is only the values of the first (from the wall) velocity maxima that noticeably differ.

Figure 5 shows $u_w$ values calculated from formula (19) with the use of the averaged porosity. It is seen that the increase in the averaging scale makes $u_w$ substantially smaller. Figure 6 compares the solutions of the Brinkman equation (17) and the generalized Brinkman equation (20). A trend for decreasing relative-velocity variations with growth in $Re_0$ is observed. It is noteworthy that this fact has long been known in the literature and it is common practice to use it for explanation of the significant difference in the so-called effective and true coefficients of interphase heat and mass exchange in stationary granular beds [7].

**Comparison with Experimental Data.** Figures 3, 4, and 6 compare the calculated and experimental [8] values of the local gas velocity in the wall region that have been obtained using a laser Doppler velocimeter. Despite the limited number of experimental points, we observe their good agreement as far as both sticking and slip on the wall are concerned for $k = 0.1$; this confirms the correctness of the procedure proposed for calculation of the distributions of the liquid (gas) velocities that is based on the use of the generalized Brinkman equation (13). The experimental $u/u_0$ values and those calculated from the Brinkman equation (17) and the generalized Brinkman equation (20) are

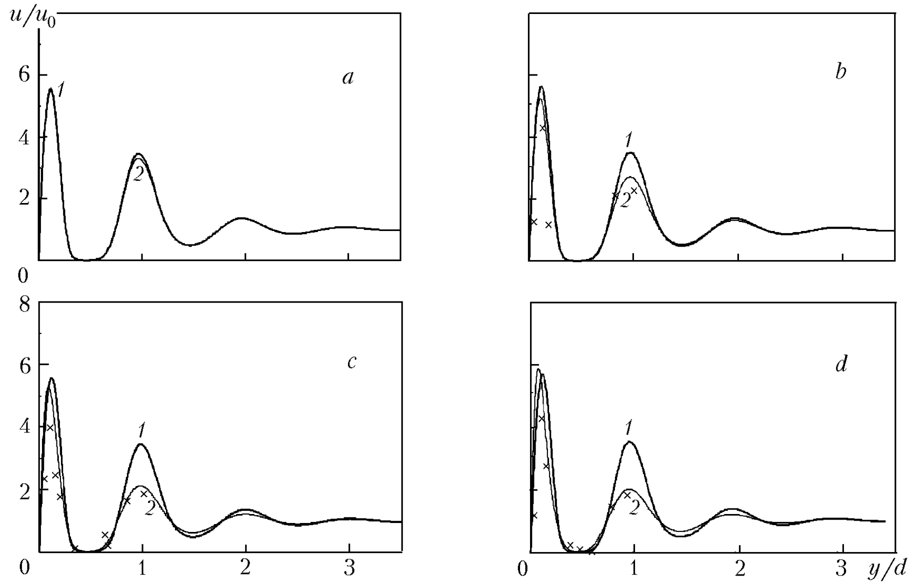


Fig. 6. Wall-velocity distribution under sticking conditions on the wall: a) $Re_0$ = 1, b) 10, c) 50, and d) 100; 1) according to the solution of the Brinkman equation (17); 2) according to the solution of the generalized Brinkman equation (20); points, experimental data [8].

compared in Fig. 6. As is seen, the generalized Brinkman equation, reflecting the evolution of the relative-velocity distribution with change in the $Re_0$ number, enables us to describe experimental data much better than the classical Brinkman equation.

**Conclusions.** Based on a comparative analysis of the existing models of filtration in a stationary granular bed, we have proposed a generalized Brinkman equation (13). It represents a superposition of the well-known Ergun equation and the equation of viscous flow in a channel. It has been shown that, with allowance for the actual porosity distribution in the wall zone, the generalized Brinkman equation enables one to describe the regularities of filtration in the granular bed near the wall in different regimes of flow (including nonlinear filtration).

This work was carried out with support from the Belarusian Republic Foundation for Basic Research (grant T05-042).

## NOTATION

$d$, particle diameter, m; $k$, averaging parameter in (21); $P$, pressure, kg/(m·sec$^2$); $R$, tube radius, m; $r$, radial coordinate, m; $r' = r/R$; $Re_0 = u_0 d/\nu_f$, Reynolds number; $u$, velocity of filtration of a liquid (gas), m/sec; $y = R-r$, m; $x$, longitudinal coordinate, m; $\alpha_1$ and $\alpha_2$, coefficients in (6), 1/sec and 1/m; $\varepsilon$, porosity; $\mu_f$, dynamic viscosity of a medium, kg/(m·sec); $\nu_f$, kinematic viscosity of a medium, m$^2$/sec; $\rho_f$, density of a medium, kg/m$^3$. Subscripts: f, medium (gas or liquid); 0, in the bed's core; w, wall.

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